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CONJUGATED CONVECTIVE HEAT TRANSFER PROBLEMS FOR VISCOPLASTIC FLUIDS IN PLANE-PARALLEL CHANNELS

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Abstract—Although the mathematical description of convective heat transfer in conjugated formulation has found the general use to date, only a few of the publications present calculations for nonlinear-viscous 'power-law' fluids. But even these lack a general analysis of the effect of the rheological factor on heat transfer. The present paper deals with the study of the conjugated heat transfer problems for nonlinear viscoplastic fluid flows in plane-parallel channels. Basic criteria of the conjugated problems are established. An analysis is made of the possible simplifications which can be incorporated into the mathematical formulation of the problem depending on the relationship between the properties of the fluid and the channel wall material.

NOMENCLATURE

a_1,	thermal diffusivity of fluid;
b,	channel wall thickness;
E,	parameter of conjugation;
h,	channel halfwidth;
l,	characteristic dimension of a change in longitudinal direction;
P,	pressure;
Pe,	Peclet number;
T_1, T_2,	temperature of fluid and channel wall, respectively;
$T_e(x)$, $q_e(x)$,	prescribed temperature and heat flux at the external wall of the channel, respectively;
T^*,	temperature due to viscous dissipation;
u,	fluid velocity;
u_0,	axial fluid velocity;
u_c,	velocity of the core;
x,	longitudinal coordinate;
y,	transverse coordinate;
α_e,	heat-transfer coefficient at the external wall of the channel;
$\tilde{\gamma}$,	$= f(\tau)$, rheological law;
$\tilde{\gamma}_s$,	velocity gradient at the wall;
δ_1,	thermal layer thickness in fluid;
λ_1, λ_2,	thermal conductivity of fluid and wall, respectively;
Δ,	size of the region with linear velocity distribution;
μ,	plastic viscosity;
τ,	shear stress;
τ_w,	shear stress at the wall.

INTRODUCTION

MATHEMATICAL analysis of convective heat transfer in a conjugated formulation is finding ever increasing use today [1]. This approach involves simultaneous determination of temperature fields in the body and the fluid flow around it with account for the relationship between them. The conditions of conjugation at the solid–fluid interface in the flow are determined by the processes occurring on the body surface. In the simplest case this is continuity of temperature and heat fluxes at the said interface. The majority of the available publications deal with the conjugated problems for Newtonian fluid flows and only a few of the works present calculations carried out for nonlinear-viscous 'power-law' fluids. However, these publications too fail to provide the general analysis of the rheological factor effect on heat transfer, nor do they establish the criteria which would have allowed one, prior to the solution of a thermal problem, to determine the possibility for the conjugated formulation to be replaced by a more simple one, neither reveal the regularities in variation of the heat transfer characteristics depending on the basic parameters of the process.

This work is concerned with the study of conjugated heat transfer problems for nonlinear viscoplastic fluid flows in plane-parallel channels. The physical properties of the fluid and of the wall material are assumed to be constant, but a number of results are also extended to the case of temperature-dependent rheological properties. A developed temperature field far from the inlet and outlet sections is considered under the given thermal conditions on the external surfaces of the channel walls (symmetric boundary conditions of the first, second or third kind) which are identical on the both sides of the channel. The analysis carried out yields a number of familiar results [2], but, being performed from a consecutive point of view, it allows one to define clearly the limits of applicability of different approximations.

As is known, the linear law of distribution of shear stresses across a plane-parallel channel is valid for a developed flow of any fluid and the velocity profile is determined directly from the following rheological equation

$$\frac{du}{dy} = f(\tau), \quad \tau = \tau_w \frac{h-y}{h}, \quad \tau_w = -\frac{dp}{dx}h$$

where $2h$ is the channel width, y is reckoned from the wall.

The mathematical formulation of the conjugated problem is reduced to the equations of thermal energy transport for the fluid and the channel wall

$$\frac{u(y)}{a_1}\frac{\partial T_1}{\partial x} = \frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} + \frac{\tau f(\tau)}{\lambda_1} \quad (1)$$

$$0 = \frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2}. \quad (2)$$

The condition of symmetry of the temperature field about the channel axis

$$\left.\frac{\partial T_1}{\partial y}\right|_{y=h} = 0 \quad (3)$$

and the condition of conjugation on the inner channel surface

$$\lambda_1 \left.\frac{\partial T_1}{\partial y}\right|_{y=0} = \lambda_2 \left.\frac{\partial T_2}{\partial y}\right|_{y=0}, \quad T_1\Big|_{y=0} = T_2\Big|_{y=0} \quad (4)$$

are supplemented with the conditions of the first, second or third kind on the external surface, respectively

$$T_2\Big|_{y=-b} = T_e(x); \quad -\lambda_2 \left.\frac{\partial T_2}{\partial y}\right|_{y=-b} = q_e(x)$$

$$-\lambda_2 \left.\frac{\partial T_2}{\partial y}\right|_{y=-b} + \alpha_e\left(T_2\Big|_{y=-b} - T_e\right) = q_e(x). \quad (5)$$

Here, $T_e(x)$ and $q_e(x)$ are the prescribed thermal conditions on the external channel walls; α_e is the heat-transfer coefficient at the external channel surface; a_1, λ_1, λ_2 the thermophysical properties of the fluid and the wall; b the wall thickness.

An important feature of the conjugated statement is the second derivative with respect to x in the heat conduction equation for the wall. As a result, thermal perturbation which originates in the flow can, under certain conditions (even at $Pe \gg 1$), propagate upstream along the channel walls, while in a non-conjugated statement, 'thermal perturbation' at $Pe \gg 1$ is only carried away by the fluid downstream of the flow.

Owing to the linearity of the problem (1)–(5), the temperature field can be sought in the form of the superposition $T(x, y) + T^*(x, y)$. Here $T^*(x, y)$ is the temperature field due to viscous dissipation in the fluid and constant components of q_e and T_e in the boundary conditions (5); $T(x, y)$ is the field produced by $q_e(x)$ or $T_e(x)$ which vary along the channel.

Let us take a particular solution of the problem (1)–(5) in the form

$$T_i^*(x, y) = Ax + B_i(y) + C, \quad i = 1, 2 \quad (6)$$

where

$$B_1 = \frac{h^2}{\tau_w^2}\left[\frac{Ah}{\tau_w a_1}\frac{\tau^2}{2}\int_0^{\tau_w} f(\xi)\,d\xi + \left(\frac{Ah}{\tau_w a_1} - \frac{1}{\lambda_1}\right)\tau\int_0^{\tau}\xi f(\xi)d\xi - \left(\frac{Ah}{2\tau_w a_1} - \frac{1}{\lambda_1}\right)\int_0^{\tau}\xi^2 f(\xi)\,d\xi\right]$$

$$B_2 = B_1(\tau_w) - \frac{\lambda_1}{\lambda_2}\frac{\tau_w}{h}\left.\frac{dB_1}{d\tau}\right|_{\tau=\tau_w} y$$

A and C are the constants that determine the temperature along the channel axis, since $B_1(h) = 0$.

Equation (6) allows one to construct the solution of the conjugated problem under different boundary conditions (5) for the constant components of T_e or q_e. Under the boundary conditions of the first and the third kinds, one obtains, respectively

$$A=0; C=T_e + \frac{h^2}{\tau_w^2\lambda_1}\int_0^{\tau_w}\left[\left(1+\frac{\lambda_1 b}{\lambda_2 h}\right)\tau_w\xi - \xi^2\right]f(\xi)\,d\xi$$

$$A=0; \quad C=T_e + \frac{h^2}{\tau_w^2\lambda_1} \times \int_0^{\tau_w}\left[\left(1+\frac{\lambda_1 b}{\lambda_2 h}\right)\tau_w\xi - \xi^2\right]f(\xi)\,d\xi + \frac{1}{\alpha_e}\left[q_e + \frac{h}{\tau_w}\int_0^{\tau_w}\xi f(\xi)\,d\xi\right]$$

while for the boundary condition of the second kind

$$A = \frac{\tau_w a_1}{h\lambda_1}\left[1 + \tau_w q_e\Big/h\int_0^{\tau_w}\xi f(\xi)d\xi\right]$$

where C is to be found from the boundary-value problem which describes the inlet section of the channel.

Thus, under the boundary conditions of the first and third kinds, the temperature field T^* does not change along the channel, while under the condition of the second kind, it amplifies linearly or attenuates along the channel depending on the relationship between the heat removed from the external surface of the channel wall and that released in the fluid.

The linear problem (1)–(5) relative to the field $T(x, y)$ is important for the analysis of heat transfer of fluids with temperature-dependent rheological properties. In this case, under the boundary conditions of the first and third kinds on the external wall surface, the temperature field can be represented in the first approximation as the sum $T^*(y) + T(x, y)$, where the field $T(x, y)$ is again conditioned by the variable components $q_e(x)$ and $T_e(x)$. Such an approximation is valid provided the rheological properties, $X(T)$, of the fluid satisfy the relationship

$$\left|\frac{dX}{dT}(T_1^*)\,T_1(x, y)\right| \ll \left|X(T_1^*)\right|.$$

But then, in order to calculate $T^*(y)$, one cannot make use of the simple formulae (6). It is this general situation which will be investigated hereafter.

First, consider different limiting cases in which the energy equation for fluid (1) is simplified. Let l be a characteristic dimension of a change in the thermal effect, $q_e(x)$ or $T_e(x)$. This very quantity also determines a scale of variation of temperature fields in the fluid and the wall along the channel. For a perfectly heated channel, when the fluid temperature in the cross-section changes but slightly, the quantity h should be taken as a transverse scale. By passing in (1) to the nondimensional quantities $x, y, u(y) \to \bar{x} = x/l$, $\bar{y} = y/h$, $\bar{u} = u(y)/u_0$, where u_0 is the axial fluid velocity, and expanding the relation for temperature at $u_0 h^2/a_1 l$, $h^2/l^2 \ll 1$ in a series of these parameters, one shall obtain (in dimensional quantities)

$$\frac{\partial T_1}{\partial y} = -\frac{\mathrm{d}T_s}{\mathrm{d}x}\left[\int_y^h u(y)\,\mathrm{d}y \Big/ a_1\right] + \frac{\mathrm{d}^2 T_s}{\mathrm{d}x^2}(h-y)$$

where $T_s(x)$ is the temperature of the inner channel surface.

In particular, this leads to the relationship

$$\left.\frac{\partial T_1}{\partial y}\right|_{y=0} = -\frac{Q}{2a_1}\frac{\mathrm{d}T_s}{\mathrm{d}x} + h\frac{\mathrm{d}^2 T_s}{\mathrm{d}x^2} \tag{7}$$

where

$$Q = 2\int_0^h u(y)\,\mathrm{d}y$$

is the fluid flow rate. Formula (7) applies when $Qh/2a_1 l \ll 1$. With this inequality being written in terms of the mean flow velocity, $\langle u \rangle = Q/2h$

$$\langle Pe \rangle = \frac{\langle u \rangle l}{a_1} \ll \frac{l^2}{h^2}.$$

Note, that for viscoplastic fluids, particularly those with the temperature-dependent rheological properties, the following inequality can hold

$$\langle u \rangle \ll u_0.$$

Another important limiting case is provided by the situation when one can neglect the convective component of the heat flux in the fluid. For a perfectly heated channel this approximation is realized at $\langle Pe \rangle \ll 1$.

Let us assume that the temperature field penetrates the fluid to a small depth, $\delta_1 \ll h$. It is seen directly from (1) that then $\delta_1 \sim l$ and at $l \ll h: T_1 \sim T_s(x)\exp(-y/l)$. Evaluation of the relative contribution of convective terms yields

$$\frac{l}{T_1 a_1}\int_0^h u(y)\frac{\partial T_1}{\partial x}\,\mathrm{d}y \sim \int_0^l u(y)\mathrm{d}y \Big/ a_1$$

$$= \langle Pe \rangle \frac{\langle u \rangle (l)}{\langle u \rangle} \ll 1.$$

When a stationary quasi-solid core is adjacent to the wall, the above relation is fulfilled even at large $\langle Pe \rangle$s. And if the velocity profile at the wall is correlated by the linear relation, then the non-equality acquires the form

$$\langle Pe \rangle \frac{\dot{\gamma}_s \cdot l}{2\langle u \rangle} \ll 1$$

where $\dot{\gamma}_s$ is the velocity gradient on the wall.

The question of whether or not it is justified to neglect the longitudinal heat conduction in the fluid is much more complicated. In the case of a perfectly heated channel it is directly evident from (7) that this is admissible at $\langle Pe \rangle \gg 1$. But if the fluid is heated to a small depth δ_1, then, taking into account that

$$\frac{\partial^2 T_1}{\partial y^2} \gg \frac{\partial^2 T_1}{\partial x^2}, \quad \frac{\partial^2 T_1}{\partial x^2} \sim \frac{T_1}{l^2}$$

this approximation is admissible at $\delta_1^2/l^2 \ll 1$. The value of δ_1 can be estimated with the aid of the relation

$$\delta_1 \int_0^{\delta_1} u(y)\mathrm{d}y \Big/ a_1 l \sim 1 \tag{8}$$

which is obtained from equation (1). When a region of width Δ with the linear velocity distribution is located in the vicinity of the wall, then at $\delta_1 \leqslant \Delta$ equation (8) yields

$$\delta_1 \sim (2a_1 l/\dot{\gamma}_s)^{1/3}.$$

The longitudinal heat conduction can be neglected at $(\dot{\gamma}_s l^2/2a_1)^{2/3} \gg 1$.

Many types of flows are characterized by quasi-solid zones moving with constant velocity. If $u(y) \sim u_c$ at $\Delta_1 \ll y \ll \Delta_2$, then equation (8) yields $\delta_1 \sim (a_1 l/u_c)^{1/2}$ for $\Delta_1 \ll \delta_1 \ll \Delta_2$ and the longitudinal heat conduction is insignificant at $u_c l/a_1 \gg 1$.

The above examples show that the ratio δ_1/l depends on the kind of the velocity profile and is not characterized by the single integral parameter $\langle Pe \rangle$.

With the use of equation (8) the condition for the longitudinal heat conduction to be neglected can be written as

$$\langle Pe \rangle \frac{\langle u \rangle (\delta_1)}{\langle u \rangle} \gg 1.$$

The analysis performed makes it possible to interrelate the temperature of the inner channel surface and the liquid temperature gradient on it. Using the conjugation condition (4)

$$\left.\frac{\partial T_2}{\partial y}\right|_{y=0} \sim -\frac{\lambda_1}{\lambda_2}\xi \left. T_2 \right|_{y=0} \tag{9}$$

where ξ depends on the hydrodynamic properties of the flow (Fig. 1)

$$\xi \sim \begin{cases} h/l^2 & \text{in region I} \\ \langle Pe \rangle (h/l^2) = Q/2a_1 l & \text{in region II} \\ 1/l & \text{in region III} \\ 1/\delta_1 & \text{in region IV.} \end{cases}$$

In region I ($l/h \gg 1$, $\langle Pe \rangle \ll 1$), the fluid temperature

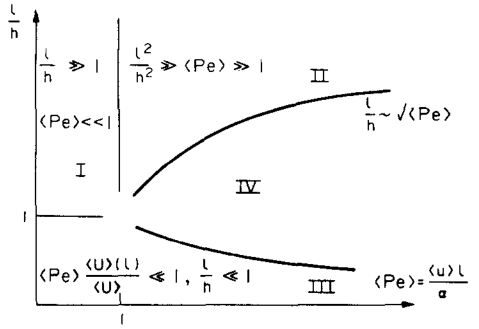

FIG. 1. Domains of the parameter l and $\langle Pe\rangle$ in which the energy equation for a fluid is simplified.

varies but slightly across the channel. The effect of fluid motion on heat transfer is negligible. In region II ($l^2/h^2 \gg \langle Pe\rangle \gg 1$), the fluid temperature also shows a slight change across the channel. But here the basic part is played by the convective heat flux. The longitudinal heat conduction is insignificant. In region III ($\langle Pe\rangle\,\langle u\rangle\,(l)/\langle u\rangle \ll 1$, $l/h \ll 1$), the temperature field penetrates the fluid to a small depth $\sim l$. The effect of fluid motion on heat transfer is negligible. In region IV, the temperature field also penetrates the fluid to a small depth δ_1 determined from (8). The longitudinal heat conduction is small if

$$\langle Pe\rangle \frac{\langle u\rangle\,(\delta_1)}{\langle u\rangle} \gg 1.$$

Let us analyse the effect of different factors on the nature of the temperature field in the conjugated problem (1)–(5). At $l \ll$ b, the temperature field produced by the specified thermal conditions on the external surface of the wall is localized within the region of length l in the vicinity of this surface. The conjugated problem should, therefore, be analyzed only for $l \gtrsim b$. Then the derivatives in the heat conduction equation for the wall are of the following order

$$\frac{\partial^2 T_2}{\partial x^2} \sim \frac{T_2|_{y=0}}{l^2},\ \frac{\partial^2 T_2}{\partial y^2} \sim \left[\left.\frac{\partial T_2}{\partial y}\right|_{y=0} - \left.\frac{\partial T_2}{\partial y}\right|_{y=-b}\right]\Big/ b.$$

With the use of equation (9) we obtain

$$\left.\frac{\partial T_2}{\partial y}\right|_{y=-b} \sim \left(1 + \frac{\lambda_2 b}{\lambda_1 \xi l^2}\right)\left.\frac{\partial T_2}{\partial y}\right|_{y=0}$$

$$T_2\Big|_{y=-b} \sim \left(1 + \frac{\lambda_1 \xi b}{\lambda_2}\right) T_2\Big|_{y=0}. \qquad (10)$$

It is evident from (10) that, depending on the value of the parameter $E = \lambda_1 \xi l/\lambda_2$, three different limiting cases take place. In addition, at

$$E \ll \frac{b}{l}\left|\frac{\partial T_2}{\partial y}(0)\right| \ll \left|\frac{\partial T_2}{\partial y}(-b)\right|$$

the temperature field varies little across the wall. The solution of the conjugated problem can therefore be approximately replaced by a successive solution of two simpler problems. First, the temperature field in the wall at the prescribed thermal conditions (5) is calculated on the outer surface and on the adiabatic inner surface. Then, the thermal problem in the fluid with the specified temperatures on the inner channel surface is solved. Note, however, that such an approach is applicable only in the case when there is a limited solution of the thermal problem for the wall with the adiabatic inner surface. At $E \gg l/b$, the value of $\partial T_2/\partial y$ changes little across the wall as well as $|T_2(0)| \ll |T_2(-b)|$. In this case, it is possible first to determine the temperature field in the wall at the prescribed thermal conditions (5) on the external surface subject to condition $T_2|_{y=0} = 0$. Then, the temperature field in the fluid with the prescribed heat flux on the inner wall surface is considered. For the intermediate region, $l/b \gg E \gg b/l$, which exists only at $b^2/l^2 \ll 1$, the changes in temperature and heat flux across the wall can be neglected. Therefore, the solution of the conjugated problem can be approximately replaced by calculation of the temperature field in the fluid, with the boundary conditions (5) being transposed from the outer to the inner wall.

Thus, it is reasonable that the conjugated problem (1)–(5) be solved only for the intermediate regions $E \sim b/l$ or $E \sim l/b$. However, even in these cases the initial formulation of the problem (1)–(5) can be somewhat simplified at $b/l \ll 1$. At $E \ll l/b$, the temperature varies slightly across the wall and the conjugated problems should be solved only for the boundary conditions of the second and third kinds on the external surface. For the first-kind conditions $T(x, y) \approx T_e(x)$.

On integrating equation (2) across the wall, neglecting the transverse change in temperature and employing the conditions (4) and (5), we shall obtain for the boundary conditions of the second and third kinds

$$b\frac{d^2 T_1|_{y=0}}{dx^2} + \frac{\lambda_1}{\lambda_2}\left.\frac{\partial T_1}{\partial y}\right|_{y=0} + \frac{q_e(x)}{\lambda_2} = 0$$

$$b\frac{d^2 T_1|_{y=0}}{dx^2} + \frac{\lambda_1}{\lambda_2}\left.\frac{\partial T_1}{\partial y}\right|_{y=0} + \frac{q_e(x)}{\lambda_2} - \frac{\alpha}{\lambda_2} T_1\Big|_{y=0} = 0. \qquad (11)$$

Equations (11) allow an approximate solution of the conjugated problem at $E \sim b/l$. When $E \gg b/l$ or $E \ll b/l$, then, ignoring the first or the second term in (11) respectively, we arrive at the cases which have already been considered. At $E \gg b/l$, the derivative $\partial T_2/\partial y$ changes little across the wall and the temperature distribution along the normal can be considered as linear.

The conjugated problem should be considered only for the boundary conditions of the first or third kind on the outer surface. For the second-kind condition $\partial T_2/\partial y \approx -q_e(x)/\lambda_2$. Using conditions (4) and (5), we shall obtain for the first- and third-kind boundary condition, respectively

$$T_1\Big|_{y=0} - \frac{\lambda_1}{\lambda_2} b \frac{\partial T_1}{\partial y}\Big|_{y=0} = T_e(x)$$
$$\alpha_e T_1\Big|_{y=0} - \lambda_1\left(1 + \frac{\alpha_e b}{\lambda_2}\right)\frac{\partial T_1}{\partial y}\Big|_{y=0} = q_e(x). \tag{12}$$

These equations make it possible to find an approximate solution for the conjugated problem at $E \sim l/b$. When $E \gg l/b$ or $E \ll l/b$, then, neglecting the respective terms in (12), we arrive at the cases considered above.

Figure 2 presents the domains of the parameter E that correspond to particular simplifications of the heat conduction equation for the wall. The quantity E has a simple physical meaning. It characterizes the ratio between the heat flux from the fluid to the wall and that which propagates along the wall at the fluid–wall boundary

$$\left|\frac{\lambda_1 \dfrac{\partial T_1}{\partial y}\Big|_{y=0}}{\lambda_2 \dfrac{\partial T_2}{\partial x}\Big|_{y=0}}\right| \sim \frac{\lambda_1}{\lambda_2}\xi l = E.$$

In order to estimate the validity of the qualitative analysis performed, accurate analytical solutions have been found for the system (1)–(5) at $q_e(x) = q_0 \exp[i(x/l)]$ and $T_e(x) = T_0 \exp[(i(x/l)]$ for four velocity profiles that corresponds to the flow of three limiting media. The constitutive equations for these media are obtained from the generalized equation [3]

$$\tau^{1/n} = \tau_0^{1/n} + (\mu\dot{\gamma})^{1/m}$$

where

$$\tau_0 = 0, \frac{n}{m} = 0; \tau_0 = 0, \quad \frac{n}{m} = \infty;$$
$$n = 1, m = \infty; \tau_0 = 0, \frac{n}{m} = 1.$$

Asymptotic expansion of these solutions for the limiting cases prove the validity of the general qualitative estimates and of possible simplifications which can be incorporated into the mathematical formulation of the conjugated problems for non-Newtonian fluids.

The analysis shows that a very simple conjugated problem is that for a perfectly heated fluid when $l^2/h^2 \gg \langle Pe \rangle \gg 1$ (Fig. 1). In this case the conjugated formulation reduces to the boundary-value problem for equation (2) subject to the boundary condition on the inner surface

$$\left(\frac{\partial T_2}{\partial y} + E\frac{\partial T_2}{\partial x}\right)_{y=0} = 0, \quad E = \frac{\lambda_1 Q}{2\lambda_2 a_2}$$

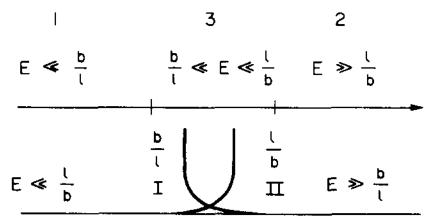

FIG. 2. The values of the parameter E at which simplifications of the conjugated problem are admissible. At $b/l \ll 1$, equations (11) apply in region I and equations (12) in region II.

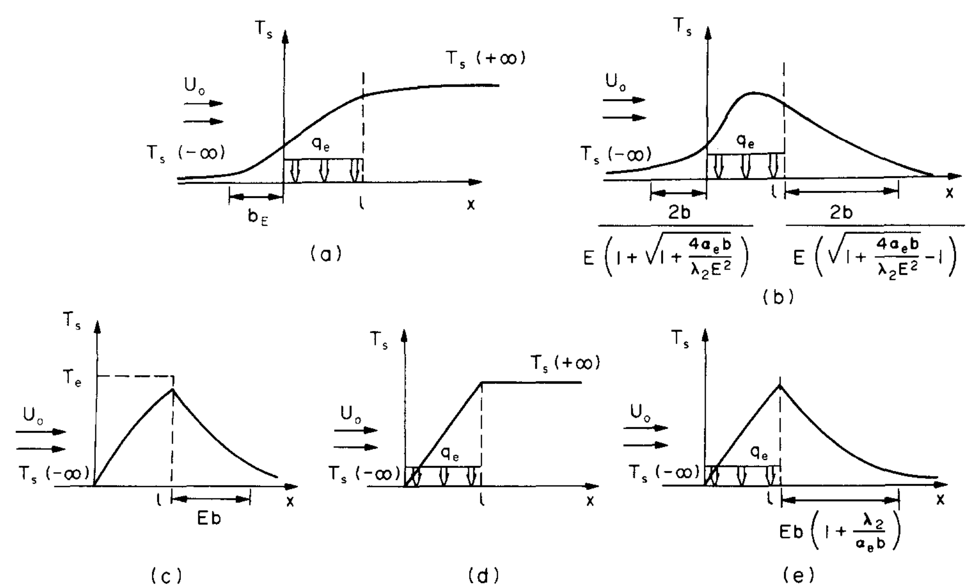

FIG. 3. Temperature distribution along the length of the channel on the inner wall surface due to thermal perturbation distributed uniformly over the segment l on the external wall surface at $E \ll l/b$ (a, b) and at $E \gg b/l$ (c, d, e) for the boundary conditions of the first (c), second (a, d) and third (b, e) kinds.

which is obtained from equations (4) and (7). This is the well-known Gilbert boundary-value problem [4]. For the first- and second-kind boundary conditions on the external surface it is reduced, with the aid of a special conformal transformation [5], to the Dirichlet problem. When $b^2/l^2 \ll 1$, then, depending on the value of E (Fig. 2), one can employ either equation (11) or (12). The wall temperature $T_s(x)$ is then determined from ordinary second- or first-order differential equations. Figure 3 shows an example of temperature variation along the channel due to thermal perturbation distributed uniformly over the segment of length l $q_e = q_0[1(x) - 1(x - l)]$ or $T_e = T_0[1(x) - 1(x - l)]$.† It is seen from this figure that at $E \ll l/b$ the effect of thermal perturbation propagates upstream of the flow.

The analysis of the above simple examples shows that the proposed methods for simplified solution of the conjugated convective heat transfer problem actually provide the basic terms of the asymptotic expansions for the temperature field at different relationships between the quantities E, $\langle Pe \rangle$, l/h and b/l.

†Calculations are given in [6].

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PROBLEMES DE COUPLAGE AVEC TRANSFERT CONVECTIF POUR DES FLUIDES VISCOPLASTIQUES DANS UN CANAL ENTRE PLANS PARALLELES

Résumé—Bien que la description mathématique du transfert thermique convectif dans une formulation de couplage est généralement connue, peu de publications présentent des calculs pour des fluides à viscosité en loi de puissance. Il manque une analyse générale de l'effet du facteur rhéologique sur le transfert de chaleur. On étudie ici les problèmes des transferts thermiques couplés pour des écoulements entre plans parallèles de fluides viscoplastiques non linéaires. On établit des critères fondamentaux. On considère les simplifications possibles qui peuvent être introduites dans la formulation mathématique du problème selon la relation entre les propriétés du fluide et le matériau constituant la paroi du canal.

GEKOPPELTER KONVEKTIVER WÄRMETRANSPORT VISKOPLASTISCHER FLUIDE IN EBENEN PARALLELEN KANÄLEN

Zusammenfassung—Obwohl die mathematische Beschreibung des konvektiven Wärmeübergangs in gekoppelter Formulierung derzeit allgemein bekannt ist, behandeln nur einige der Veröffentlichungen Berechnungen für nichtlinear viskose Flüssigkeiten. Aber selbst bei diesen fehlt eine allgemeine Behandlung des Einflusses des Rheologie-Faktors auf den Wärmeübergang.

Die vorliegende Arbeit befaßt sich mit der Untersuchung des gekoppelten Wärmetransportproblems für nichtlineare viskoplastische Fluidströmungen in ebenen parallelen Kanälen. Grundlegende Kriterien des gekoppelten Problems werden aufgestellt. Eine Untersuchung über mögliche Vereinfachungen wird durchgeführt, welche in Abhängigkeit von der Beziehung zwischen den Eigenschaften des Fluids und dem Material der Kanalwände bei der mathematischen Beschreibung des Problems getroffen werden können.

СОПРЯЖЕННЫЕ ЗАДАЧИ КОНВЕКТИВНОГО ТЕПЛООБМЕНА ДЛЯ ВЯЗКОПЛАСТИЧНЫХ ЖИДКОСТЕЙ

Аннотация — Хотя математическое описание конвективного теплообмена в сопряженной постановке получило в последнее время большое распространение, для нелинейновязких «степенных» жидкостей расчеты представлены лишь в нескольких работах. Однако и в них отсутствует общий анализ влияния реологического фактора на теплообмен. Настоящая работа посвящена исследованию сопряженных задач теплообмена при течении нелинейно вязкопластичных жидкостей в плоскопараллельных каналах. Установлены основные критерии сопряженных задач. Проведен анализ возможных упрощений, которые могут быть использованы в математической формулировке сопряженной задачи в зависимости от соотношения между характеристиками жидкости и материала стенки канала.